Spatial modeling in transportation (lock improvements and sequential congestion)

Simon P. Anderson
University of Virginia
and
Wesley W. Wilson
University of Oregon and Institute for Water Resources

Urbino, July 13, 2007

Background

- Navigation and Economics Technologies Program (NETS)
- Cost-benefit Analysis of improving locks
- Need for spatial economic models to underpin transportation demand and congestion on the waterway (and other modes, like railroads)
- Transportation complements and substitutes
- Simple congestion in sequence of bottlenecks: effects of lock improvements

Theoretical Background

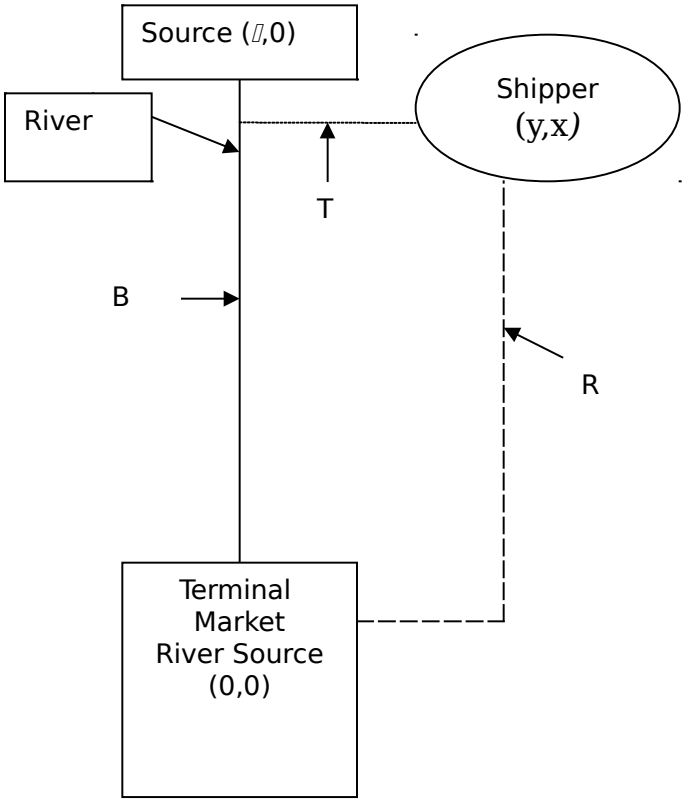
- Farmers geographically dispersed
- Truck-barge or rail (or truck)
- Lock system and congestion
- Lock by-pass
- Endogenous price of transportation services

Basic model

- Terminal market at 0
- River runs NS along y-axis
- Transport metric is Manhattan
- River rate: b
- Truck rate: t
- Rail rate: r

$$b < r < t$$

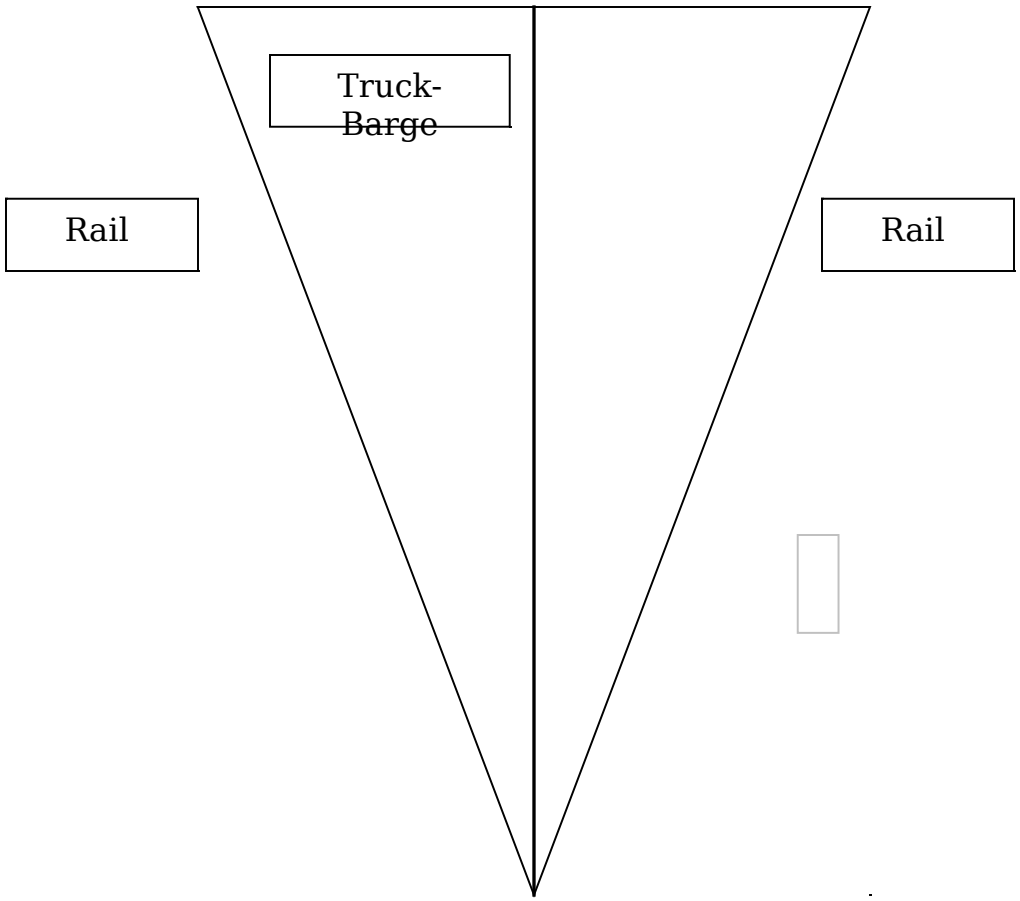
Figure 1-The Network



Truck-barge catchment area

- rx + ry > tx + by
- $y^{hat} = x(t-r)/(r-b)$
- Transport rates depend on crops etc.

Figure 2-Modal catchment areas



Fixed costs

- Now add fixed costs to shipment costs:

  For mode m:

  $F_m + md, \quad m = b, r, t$

  $F_t < F_r < F_b$

Shipment Costs (if single mode!)

Rail Movements Not Dominated

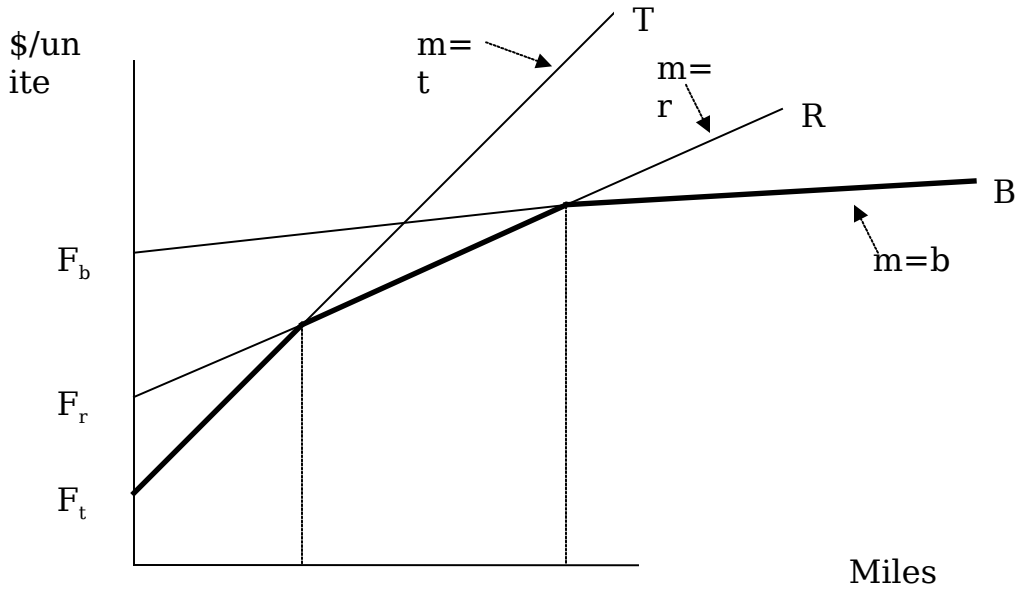
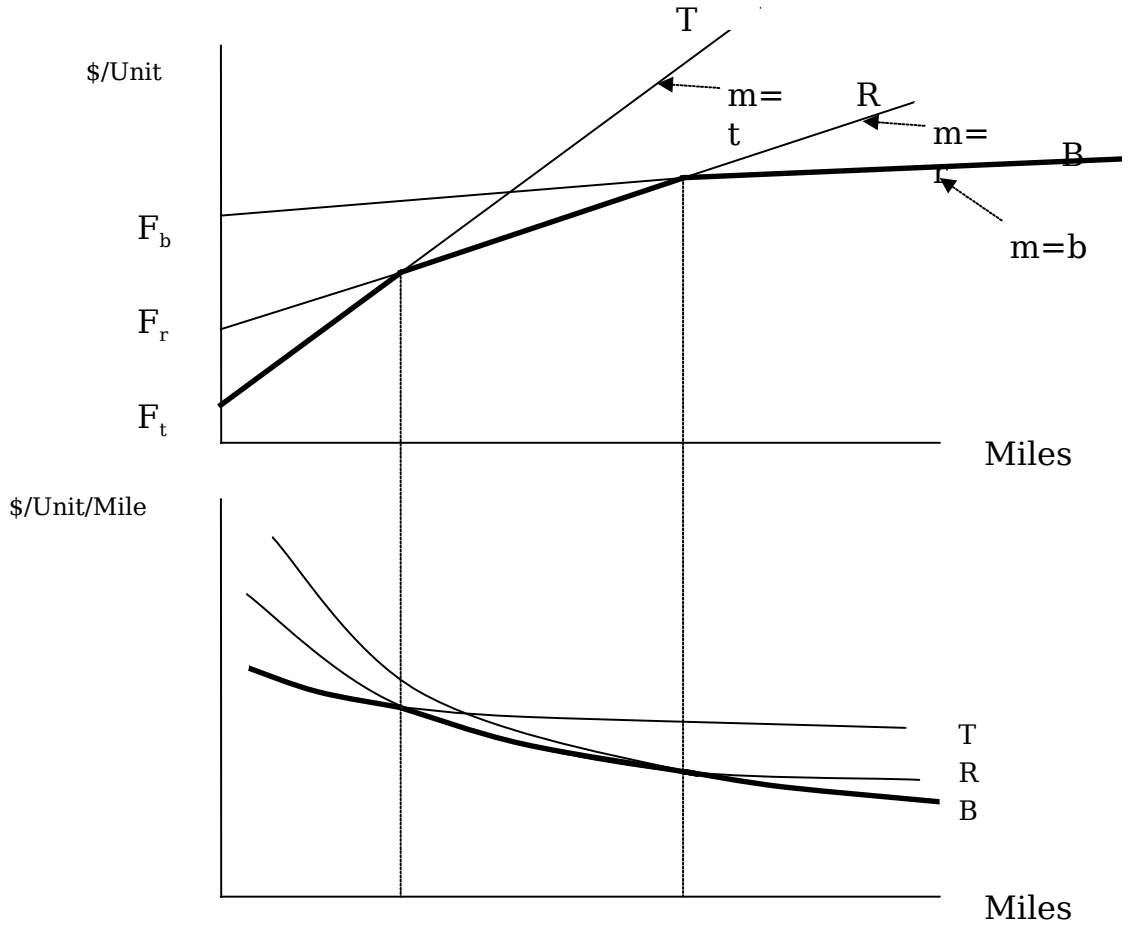


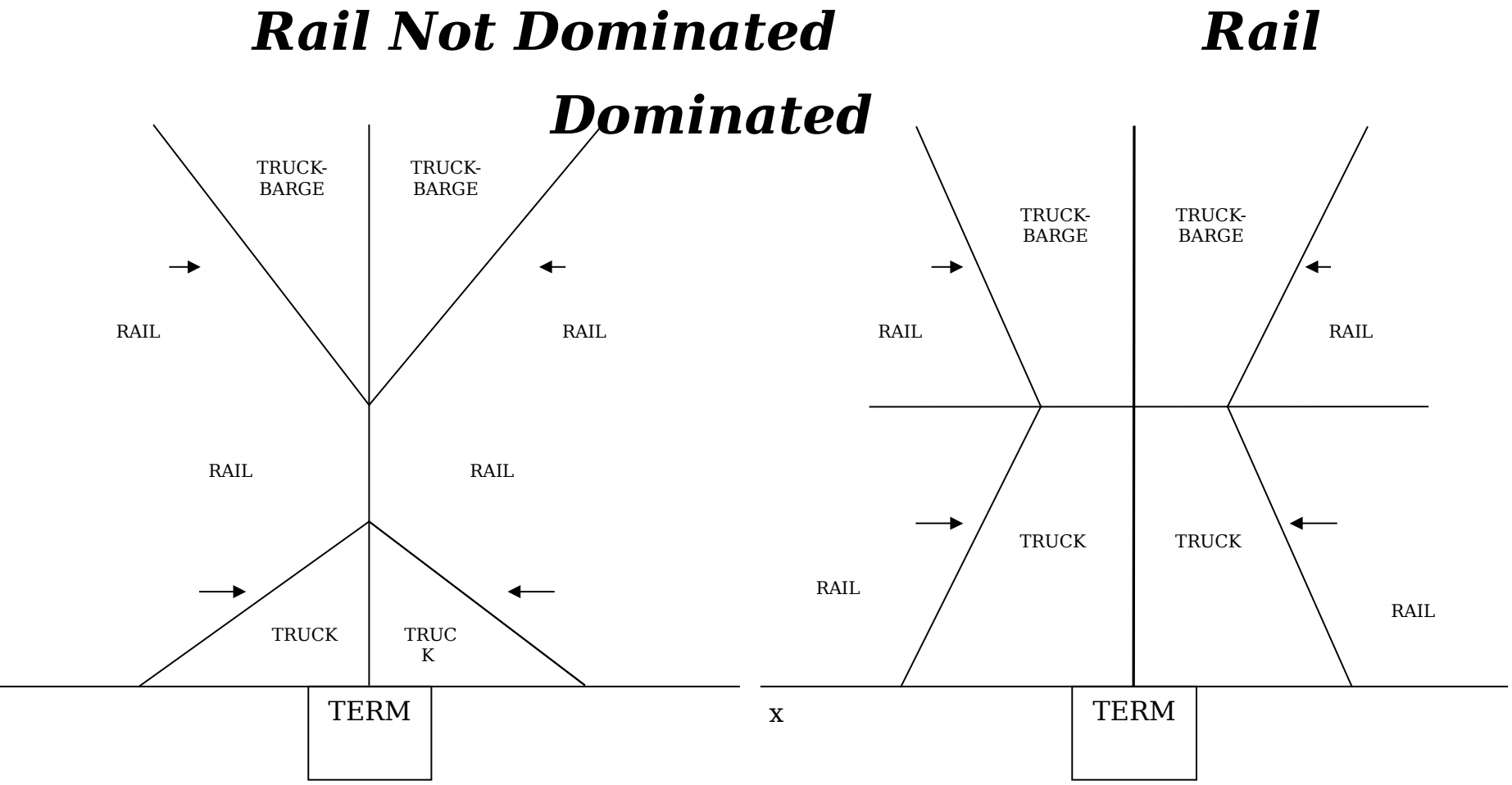
Figure 4-Rail Rate Tapers



Mode complementarities

- To use barge, must truck to river first
- Else could truck directly to final market
- Rail is a substitute to both of these options

Figures 5 and 6
Catchment areas with fixed costs



Locks

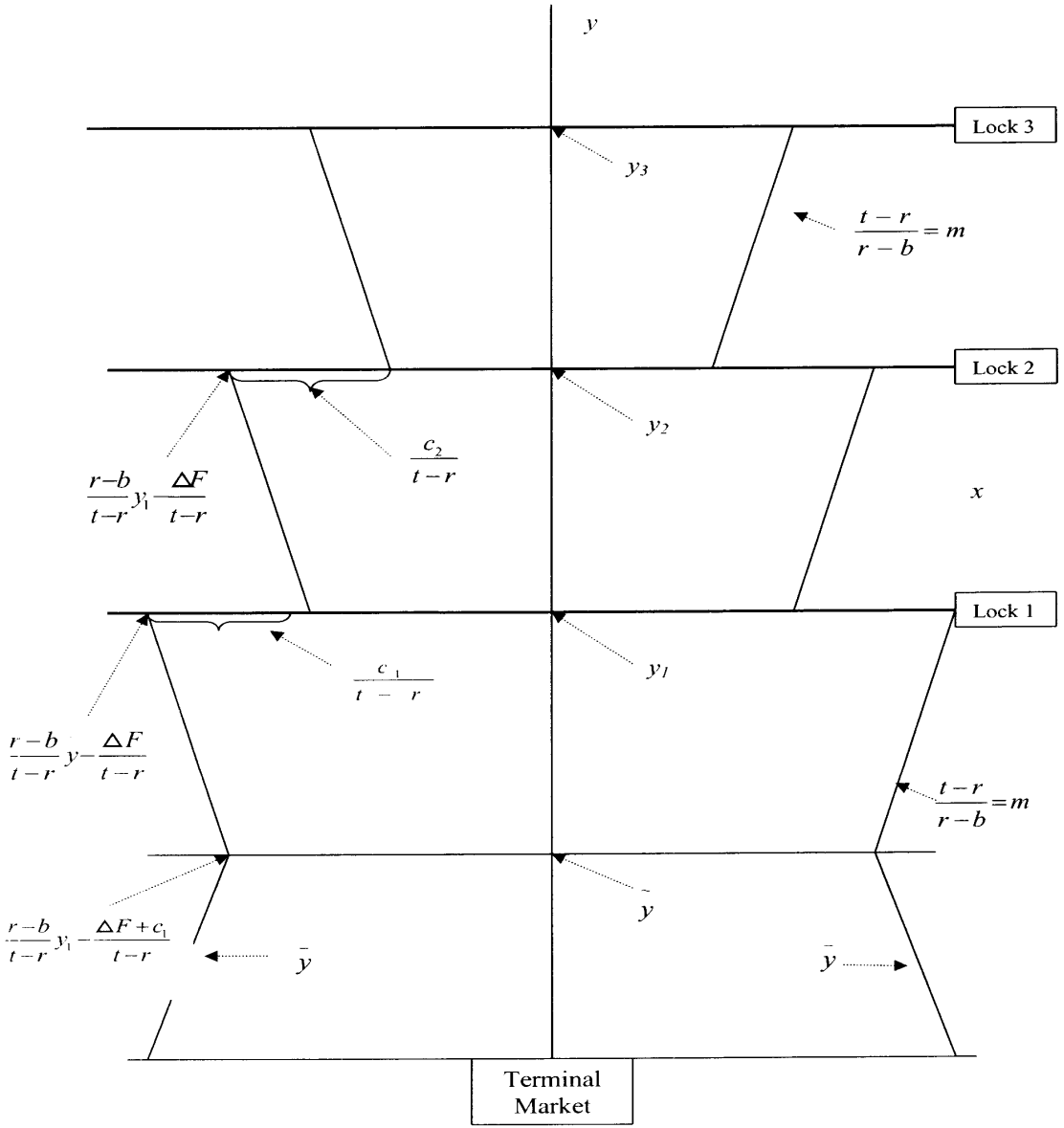
- Passing lock j costs C_j, j = 1, …, n

(cost will depend below on volume of shipping)

- Truck-barge used from (y,x) if:
- $F_r + rx + ry > F_t + F_b + tx + by + \Sigma^j C_i$

- Barge mode takes a “hit” at lock levels

Figure 7. Fixed Costs and Locks.

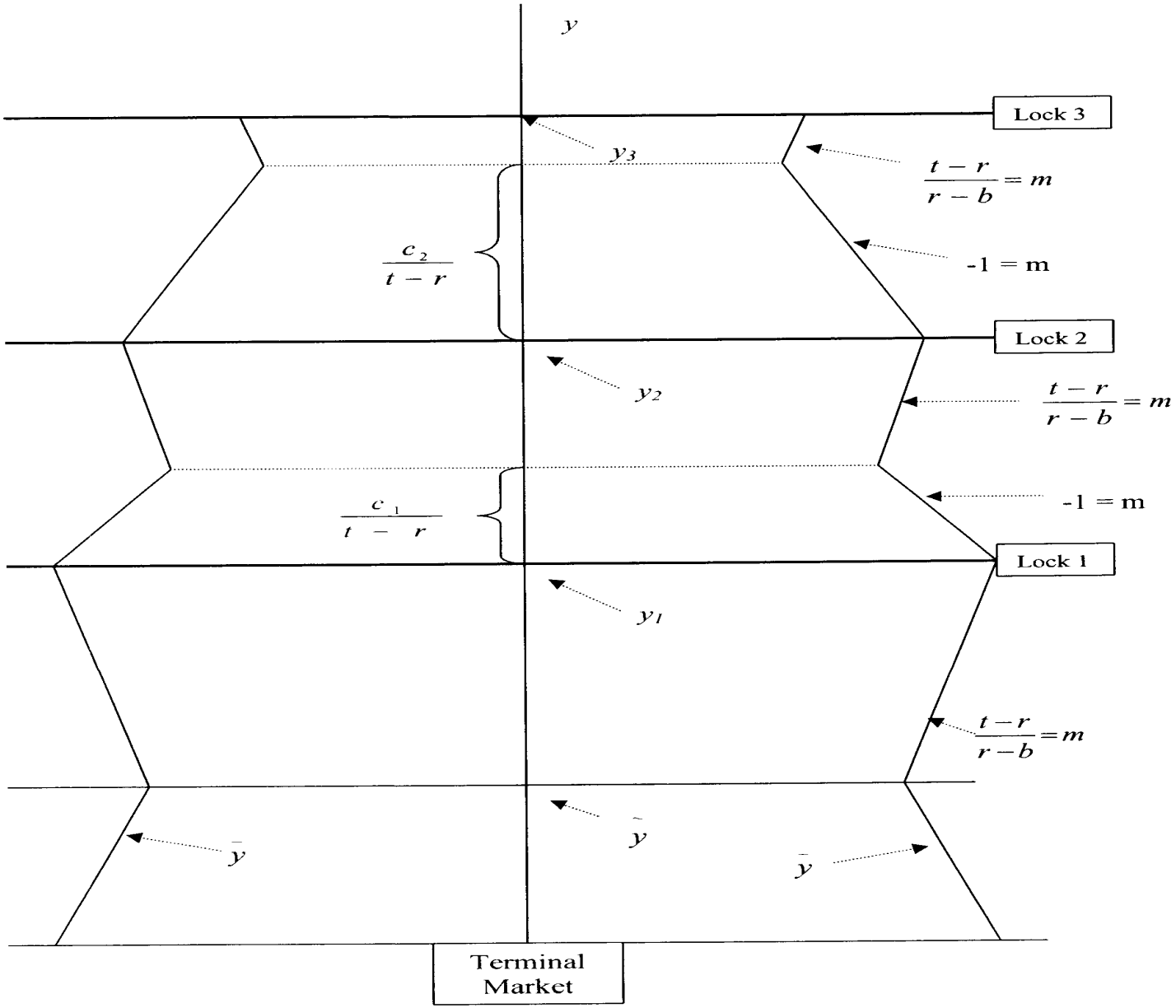


Lock by-pass

- Possible to by-pass one or several locks

- Use truck down to below the lock (enter at a river terminal)

Now advantage of rail *falls* closer to lock


y
Lock 3
y_3
$\frac{t-r}{r-b}=m$
$\frac{c_2}{t-r}$
-1 = m
Lock 2
y_2
$\frac{t-r}{r-b}=m$
$\frac{c_1}{t-r}$
-1 = m
Lock 1
y_1
$\frac{t-r}{r-b}=m$
$\bar{y}$
$\tilde{y}$
$\bar{y}$
Terminal Market

Continuity of lock demands

- Note catchment areas are continuous functions of congestion costs
- Hence shipping levels through locks are continuous
- The same holds when we allow for multiple lock by-pass (there is no “zap” price/indifference plateau where all demand suddenly shifts)

Congestion

- Depends on all shipping through lock, i.e., from all points up-river
- More traffic at locks lower down
- Single lock case: traffic depends on cost
- Cost depends on traffic. Equilibrium as FP
- Multi-lock case follows similar logic

Congestion with multiple locks

- Cost at Lock n depends on traffic emanating above it
- Traffic above Lock n depends on costs at all lower locks
- Cost at Lock n-1 depends on traffic from above n and between n-1 and n; traffic entering between n-1 and n depends on $C_1 \ldots C_{n-1}$
- Cost depends on traffic above; traffic depends on costs lower down

Existence

- Brouwer: cts mapping from a compact, convex set has a Fixed Point
- Assume D’s and C’s are cts and finite
- D’s determine C’s determine D’s … i.e., maps “old” D’s into new D’s in a cts fashion.
- Hence a fixed point exists (hence equilibrium)

Equilibrium uniqueness

- Suppose there were another.

  Suppose $D_n' < D_n \Rightarrow C_n' < C_n$

- Then $\Sigma C_j' > \Sigma C_j$ for $j < n$ (to have $D_n' < D_n$)

- Then $D_{n-1}' < D_{n-1} \Rightarrow C_{n-1}' < C_{n-1}$ etc. Hence a contradiction.

- There exists a unique solution

Improving a lock j

1. D_1 must go up

Suppose not:

$D_1 \downarrow \Rightarrow C_1 \downarrow$

$\Rightarrow \quad D_1 - D_2 \uparrow$

(more entering between locks 1 and 2)

$\Rightarrow \quad D_2 \downarrow$ (to have the original $\Rightarrow D_1 \downarrow$)

$\Rightarrow \quad D_2 \downarrow \Rightarrow C_2 \downarrow$

Hence all costs below j would decrease, so a contradiction

$C_1 < C_{n-1}$ etc $\uparrow \quad \downarrow$

Improving a lock j

Hence D_1 must go up

Then

$D_1 \uparrow \Rightarrow C_1 \uparrow$

$\Rightarrow \quad D_1 - D_2 \downarrow$

(less entering between locks 1 and 2)

$\Rightarrow \quad D_2 \uparrow$ (to have the original $\Rightarrow D_1 \uparrow$)

$\Rightarrow \quad D_2 \uparrow \Rightarrow C_2 \uparrow$

All costs below j increase … new demands decrease

j doesn’t “overshoot”: its costs still lower

All costs above j increase (locally) but totals fall so there is more traffic from all points above j

Comparative static properties

- Improve locks
- Suppose a lock in the middle is improved
- More traffic coming through from above: more congestion at higher locks. More development higher (larger catchment).
- More traffic lower down (more congestion)
- Less new traffic joining the river lower down

- Hence: more traffic at all other locks
- More production upstream, less production downstream

Barge shipping rates

- Suppose the number of barges is fixed

- The model above determines the equilibrium demand price for barge services
- Hence, with supply, can determine the equilibrium price

Conclusions

- Spatial Equilibrium with modal choice
- Series of congested points – existence and uniqueness of solution
- Improving a lock improves overall flow, but increases congestion downstream from the improvement while decreasing it upstream.
- More development further out, contraction of activity further in
- Analogy to commuter traffic
- Endogenous price of transport mode
- Basis for cost-benefit analysis
- Model can be readily calibrated

Future research directions

- bi-directional barge movements and backhauling

- Timing of shipments; speed/reliability of modes (risk); equilibrium price in final market and mode choice